

# Does nuclear matter bind at large $N_c$ ?

Luca Bonanno and Francesco Giacosa

*Institut für Theoretische Physik  
Johann Wolfgang Goethe - Universität  
Max von Laue-Str. 1 D-60438 Frankfurt, Germany*

---

## Abstract

The existence of nuclear matter at large  $N_c$  is investigated in the framework of effective hadronic models of the Walecka type. This issue is strongly related to the nucleon-nucleon attraction in the scalar channel, and thus to the nature of the light scalar mesons. Different scenarios for the light scalar sector correspond to different large  $N_c$  scaling properties of the parameters of the hadronic models. In all realistic phenomenological scenarios for the light scalar field(s) responsible for the attraction in the scalar channel it is found that nuclear matter does not bind in the large  $N_c$  world. We thus conclude that  $N_c = 3$  is in this respect special: 3 is fortunately not large at all and allows for nuclear matter, while large  $N_c$  would not.

---

## 1. Introduction

The large  $N_c$  limit constitutes a well-defined and systematic theoretical framework to address fundamental questions of QCD [1, 2, 3]. Recently, basic properties of the QCD phase diagram for  $N_c \gg 1$  have been presented in Ref. [4] and further investigated in Refs. [5, 6, 7, 8] and refs. therein.

In this work we address the following question: does nuclear matter bind for  $N_c \gg 1$ ? It is interesting to investigate whether the existence of nuclear matter is a special phenomenon of  $N_c = 3$  or it is a general feature independent on the number of colors.

It is not possible to answer this question by using QCD. In fact, the latter is not solvable, not even in the large  $N_c$  limit. Thus, the only way to study nuclear matter at large  $N_c$  is to use an effective Lagrangian. The most convenient choice for this purpose is the Walecka model [9], which has been widely used for studies at nonzero density. In fact, although this model is naive, we use it for several reasons: (i) Regardless of the real model describing nature, nuclear matter saturation can be always described by using an effective Walecka-like model including attractive scalar interactions and repulsive vector interactions. This means that the (unfortunately unknown) correct chiral model for low-energy hadrons must reduce, for densities close to nuclear matter and for small temperature, to a Walecka-like model. (ii) Although the Lagrangian does not embed

important symmetries of QCD such as chiral symmetry and scale invariance, we are limiting our study to nuclear matter. This is a regime of baryon densities at which these symmetries are strongly broken. (iii) The Walecka model has the advantage of being simple, allowing a direct understanding of the large  $N_c$  behavior of nuclear matter.

For  $N_c = 3$  the couplings are fixed to recover the usual nuclear density properties: saturation at  $\rho_0 = 0.16 \text{ fm}^{-3}$  and an energy per nucleon  $E/A = -16$  MeV. We then rescale the couplings with appropriate powers of  $N_c$  and perform the study at higher  $N_c$  values: in this way it is possible to check if nuclear matter still exists when increasing  $N_c$ .

We shall find that the binding of nuclear matter at large  $N_c$  strongly depends on the nature of the lightest scalar resonance(s) [10, 11]. In the (old-fashioned) assignment in which the lightest scalar resonance  $f_0(600)$  is predominantly a quark-antiquark state, nuclear matter binds indeed at each  $N_c$ . Moreover, the binding energy increases for increasing  $N_c$ .

However, the quark-antiquark scenario is regarded as unfavored by most recent works on the low-energy scalar sector [11, 12, 13, 14]. We thus study alternative assignments for the scalar resonances, which are in agreement with the phenomenology. For instance, the so-called tetraquark scenario can describe well the properties of the light scalar states below 1 GeV, see the original work of Ref. [15] and further investigations in Refs. [16, 17, 18, 19]. Interpreting  $f_0(600)$  as a tetraquark state, and taking into account the correct large  $N_c$  limit of these objects (Sec. 2.4), it is found that nuclear matter does *not* bind for  $N_c \gg 3$ , and indeed ceases to exist already for  $N_c = 4$ .

We also repeat our analysis for other assignments in the scalar sector: (i) Even if the light scalar states are predominantly tetraquarks, scalar quarkonium states must exist. For this reason we study an enlarged scenario in which, besides the resonance  $f_0(600)$  interpreted as predominantly tetraquark, a second scalar field, identified with the resonance  $f_0(1370)$ , is added and interpreted as predominantly quarkonium. This scenario is in agreement with the outcome of Ref. [20], where two scalar fields with similar masses are needed in order to describe nucleon-nucleon scattering data. (ii) The role of two-pion-exchange (TPE) processes can be important for nuclear matter [21]. We thus test the scenario in which a scalar field describes effectively the TPE attraction in the scalar-isoscalar channel. (iii) The assignment in which the lightest scalar state  $f_0(600)$  is interpreted as a glueball is investigated. (iv) The lightest scalar resonance  $f_0(600)$  can be also regarded as a ‘dynamically generated resonance’ emerging from pion-pion interactions [12].

It is remarkable that also in all these cases nuclear matter does *not* bind for in the large  $N_c$  limit. The non existence of nuclear matter for large  $N_c$  is thus not an artifact of a particular assignment for the light scalar states, but is a stable result as soon as the quarkonium interpretation is abandoned. Note also that, while studies of nucleon-nucleon potentials in the large  $N_c$  limit exist [22, 23], the different scaling properties for  $N_c \gg 3$  due to the non-quarkonium nature of the scalar attraction have not yet been (to our knowledge) systemically investigated.

The paper is organized as follows: in Sec. 2 we investigate the existence of nuclear matter for  $N_c \gg 1$  for different scenarios for scalar resonances and in Sec. 3 we draw our conclusions.

## 2. Nuclear matter at large $N_c$

### 2.1. The *Walecka model*

The Lagrangian of the *Walecka model* reads [9]:

$$\begin{aligned} \mathcal{L} = & \bar{\psi}[\gamma^\mu(i\partial_\mu - g_\omega\omega_\mu) - (m_N - g_\sigma\sigma)]\psi + \frac{1}{2}\partial^\mu\sigma\partial_\mu\sigma - \frac{1}{2}m_\sigma^2\sigma^2 - \frac{1}{4}F^{\mu\nu}F_{\mu\nu} \\ & + \frac{1}{2}m_\omega^2\omega^\mu\omega_\mu - V_\sigma(\sigma) , \end{aligned} \quad (1)$$

where  $\psi$  and  $\omega_\mu$  are the nucleon and the isoscalar-vector field, respectively, and  $F_{\mu\nu} = \partial_\mu\omega_\nu - \partial_\nu\omega_\mu$ . Finally,  $\sigma$  is a scalar field, usually identified with the lightest scalar resonance:  $f_0(600)$ .  $V_\sigma(\sigma)$  is a potential containing self-interaction terms of the  $\sigma$  field, which we do not consider here for simplicity.

The mean values of the scalar and vector condensates can be easily found by solving the Euler-Lagrange equations:

$$\bar{\sigma} = \left(\frac{g_\sigma}{m_\sigma}\right)^2 \rho_s , \quad \bar{\omega}_0 = \left(\frac{g_\omega}{m_\omega}\right)^2 \rho_B , \quad (2)$$

where  $\rho_s$  and  $\rho_B$  are the scalar density and the baryon density, respectively.

### 2.2. The large $N_c$ limit

We briefly summarize basic properties of hadrons in the large  $N_c$  limit. These features have been originally investigated in Refs. [1, 2] and reviewed in Ref. [3]. The following properties at large  $N_c$  have been deduced:

- Quark-antiquark meson masses have a smooth limit for large  $N_c$ :  $m_{\bar{q}q} \propto N_c^0$ . Moreover, the general  $n$ -point interaction vertex is of order  $N_c^{-(n-2)/2}$ , therefore the quark-antiquark states are non-interacting particles when  $N_c \rightarrow \infty$ .
- The baryon mass is of order  $N_c$ .
- The baryon-meson amplitude is of order  $\sqrt{N_c}$  and the baryon-baryon interaction, mediated by meson exchange, is of order  $N_c$ .
- Four-quark states do not survive in the large  $N_c$  limit, but a different limit for tetraquark states can be considered, see Sec. 2.4 for details.

### 2.3. The naive quark-antiquark scenario

We start by considering the case in which the medium-range attraction among nucleons is only mediated by the exchange of a quark-antiquark state, that can be identified with the lightest scalar resonance  $f_0(600)$ . This is the old-fashioned assignment for low-energy effective models of QCD, such as the linear  $\sigma$  model [24, 25], and the Nambu Jona-Lasinio model [26].

It should be noticed already at this stage that this assignment is unfavored by most recent studies of light scalar mesons [11, 12, 13]. Also in the updated version of the linear  $\sigma$  model with (axial)vector mesons of Ref. [14], it is found that the quark-antiquark scalar state  $\sqrt{\frac{1}{2}}(\bar{u}u + \bar{d}d)$  corresponds to the resonance  $f_0(1370)$ , rather than to the resonance  $f_0(600)$ . Nevertheless, due to its historical importance and the well-defined large  $N_c$  behavior, we first investigate this scenario.

Following the scaling properties in Sec. 2.2, we can easily deduce the following scaling laws for the masses and the couplings of the model in Eq. (1):

$$m_\sigma \longrightarrow m_\sigma ; \quad (3)$$

$$m_\omega \longrightarrow m_\omega , m_N \longrightarrow m_N \frac{N_c}{3} ; \quad (4)$$

$$g_\sigma \longrightarrow g_\sigma \sqrt{\frac{N_c}{3}} , g_\omega \longrightarrow g_\omega \sqrt{\frac{N_c}{3}} . \quad (5)$$

In order to understand if nuclear matter still exists for  $N_c > 3$ , we computed the equation of state (EoS) of cold, isospin symmetric nuclear matter, making use of the scaling relations Eqs. (3), (4) and (5). In order to reproduce the saturation, we use the following numerical values of the parameters at  $N_c = 3$ :  $g_\sigma^2/4\pi = 11.10$  with  $m_\sigma = 600$  MeV and  $g_\omega^2/4\pi = 14.37$  with  $m_\omega = 783$  MeV. The results are shown in Fig. 1, where it is clear that nuclear matter reaches saturation for all values of  $N_c$ . The reason for this property is easy to understand: since in this case  $\omega_\mu$  and  $\sigma$  are both  $q\bar{q}$  states, their couplings scale in the same way. Therefore, the balance between attraction and repulsion among nucleons, leading to saturation, does not depend on the number of colors. As a consequence of the scaling, the value of the saturation density, as shown in Fig. 2, reaches an asymptotic value of  $\sim 2.3\rho_0$  when  $N_c \rightarrow \infty$ . On the other hand, since the energy of the system is of order  $N_c$ , the binding energy per nucleon at saturation must grow linearly with  $N_c$ . This behavior is shown in Fig. 3.

In Fig. 4 we show the liquid-gas critical lines for different values of  $N_c$ . Not only the critical baryon chemical potential, but also the critical temperature grows linearly with  $N_c$ . We reach a domain of temperatures in which our present mean-field and Walecka-based study should be regarded with care. Moreover, for  $N_c$  large enough, the critical temperature would also overshoot the deconfinement temperature  $T_{dec} \sim \Lambda_{QCD}$ , which is a  $N_c$ -independent quantity. It is anyhow interesting to observe that in this scenario the nuclear-liquid transition line becomes longer on the  $T$ -direction, see also the discussion in Ref. [6], in

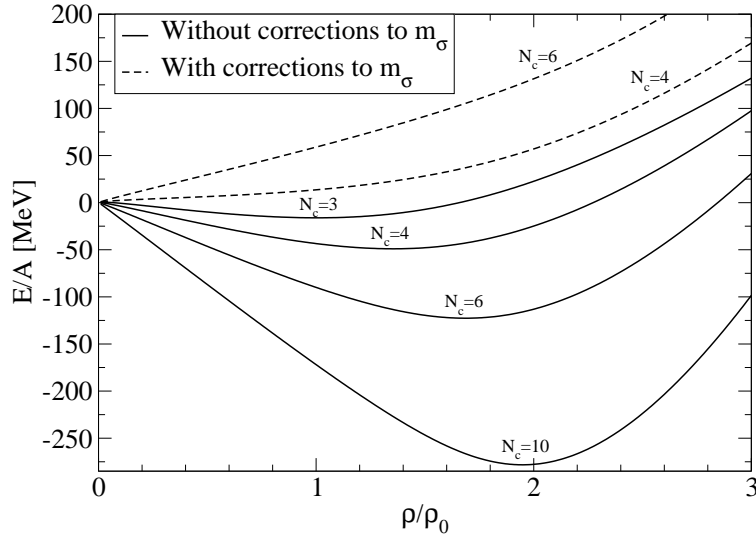


Figure 1: Binding energy per nucleon as a function of the baryon density (in units of  $\rho_0 = 0.16\text{fm}^{-3}$ ) for different values of  $N_c$ . The solid lines refer to the case in which no loop corrections to  $m_\sigma$  are considered; the dashed lines result from the inclusion of these corrections, by using Eq. (6) with  $b_\sigma = 953$  MeV.

which a Van Der Waals approach to describe the nuclear-liquid phase transition is used.

The results presented in this subsection lead – at first sight – to a strongly bound nuclear matter for large  $N_c$ . However, even in the quark-antiquark scenario for the light  $\sigma$  meson, the next-to-leading order corrections to the  $\sigma$  mass are non-negligible. Namely, the function  $m_\sigma^2(N_c)$  can be rewritten as

$$m_\sigma^2(N_c) = m_\sigma^2 + b_\sigma^2 \left( \frac{1}{3} - \frac{1}{N_c} \right). \quad (6)$$

This formula takes into account the fact that the mass for  $N_c = 3$ ,  $m_\sigma^2(N_c = 3) = m_\sigma^2$ , is reduced by meson loops with respect to the large  $N_c$  asymptotic value  $m_\sigma^2(N_c \gg 1) = m_\sigma^2 + b_\sigma^2/3$  [27]. In the case of the light  $f_0(600)$  meson this mass reduction is generated by the pion loops and is large, as the large width of this resonance confirms. Numerically, one has  $b_\sigma^2 \simeq 3(350\text{-}600 \text{ MeV})^2$ , which implies a meson-loop mass reduction of about 100-250 MeV.

Note that the same modification holds in principle also for the  $\omega$  meson, but it is negligibly small due to the very small width of this resonance. This modification can be safely omitted.

When using the more realistic Eq. (6) instead of Eq. (3) one already observes that the nuclear matter does not bind at  $N_c > 3$ . This property is shown in Fig. 1, where the dashed lines represent the binding energy per nucleon when Eq. (6) is used.

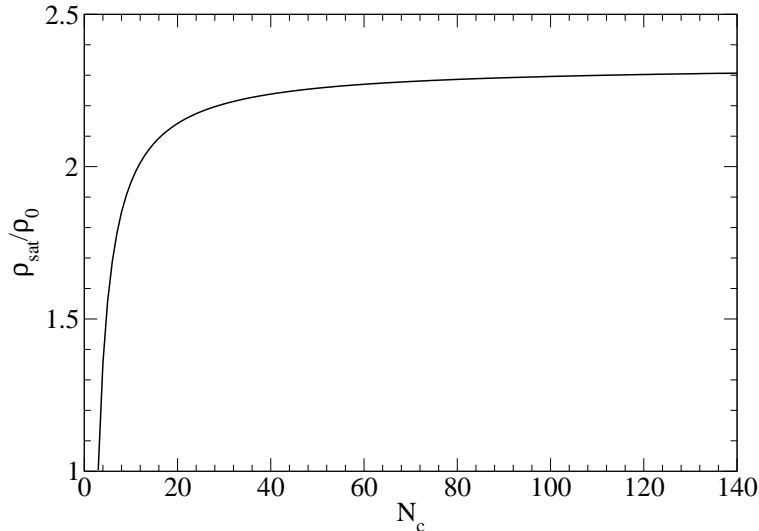


Figure 2: Baryon density at saturation (in units of  $\rho_0 = 0.16\text{fm}^{-3}$ ) as a function of the number of colors.

#### 2.4. The tetraquark scenario

The tetraquark scenario offers a consistent scheme to interpret the scalar states below 1 GeV [15, 16, 17, 18, 19]. The state  $f_0(600)$  is described as a bound state of two ‘good’ diquarks:  $f_0(600) = \frac{1}{2}[\bar{u}, \bar{d}][u, d]$ , where the commutation means anti-symmetrization in flavor space. Similarly, the color wave function of the tetraquark is given by

$$[\bar{R}, \bar{B}][R, B] + [\bar{G}, \bar{B}][G, B] + [\bar{R}, \bar{G}][R, G] , \quad (7)$$

where  $R, B, G$  refer to the three colors of the quark.

The term ‘good’ diquark [16] signalizes the flavor and color antisymmetric spinless diquark, in which a particularly strong attraction takes place [28]. The very same good diquark is also expected to be an important piece of the nucleon described as a quark-diquark bound state. It is also a basic object for color superconductivity at large density.

The relevant question for the present study is the description of a tetraquark in the large  $N_c$  limit. It is actually known that a tetraquark, which is made of two quarks and two antiquarks, does not survive in the large  $N_c$  limit. This fact was already recognized in the original works by ’t Hooft [1] and Witten [2]: instead of a tetraquark one has in the large  $N_c$  limit two ‘standard’ quark-antiquark mesons.

There is however another object which can be considered in the Large  $N_c$  limit. Considering that our tetraquark for  $N_c = 3$  consists of two good diquarks, it should be asked what is a good diquark for  $N_c > 3$ . The answer is simple: a ‘good diquark’ for  $N_c > 3$  is an object with  $N_c - 1$  quarks in a antisymmetric

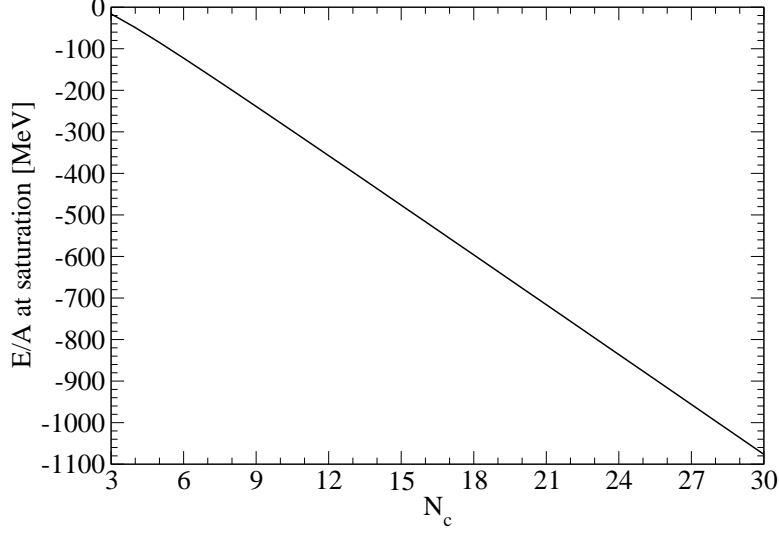


Figure 3: Binding energy per nucleon at saturation as a function of the number of colors.

color wave function:

$$d_{a_1} = \varepsilon_{a_1 a_2 a_3 \dots a_{N_c}} q^{a_2} q^{a_3} \dots q^{a_{N_c}} \text{ with } a_2, \dots, a_{N_c} = 1, \dots, N_c . \quad (8)$$

This means that the generalization of a ‘tetraquark’ for  $N_c > 3$  is a bound state of a  $(N_c - 1)$ -quark object and a  $(N_c - 1)$ -antiquark object [11, 29]:

$$\chi = \sum_{a_1=1}^{N_c} d_{a_1}^\dagger d_{a_1} . \quad (9)$$

In this way also the ‘tetraquark’ has a well-defined limit for  $N_c \gg 1$ , whose mass scales as

$$m_\chi \propto 2(N_c - 1) \sim N_c . \quad (10)$$

The interaction of the generalized tetraquark  $\chi$  with mesons and with nucleons is different:

(i) The formation of two quark-antiquark mesons requires the annihilations of  $(N_c - 3)$  quark-antiquark pairs. For this reason the amplitude for the process  $\chi \rightarrow \bar{Q}Q$ , where  $Q$  represents a quark-antiquark state, is given by  $A_{\chi \rightarrow \bar{Q}Q} \propto p^{N_c-3} \sim p^{N_c} \sim e^{-N_c}$ , where  $p$  is the annihilation probability of a quark from  $d_{a_1}$  and an antiquark from  $d_{a_1}^\dagger$ . The full decay width of the process  $\chi \rightarrow \bar{Q}Q$  as a function of  $N_c$  reads:

$$\Gamma_{\chi \rightarrow \bar{Q}Q} \sim \frac{\sqrt{\frac{m_\chi^2}{4} - m_Q^2}}{m_\chi^2} [A_{\chi \rightarrow \bar{Q}Q}]^2 \sim \frac{1}{N_c} e^{-N_c} . \quad (11)$$

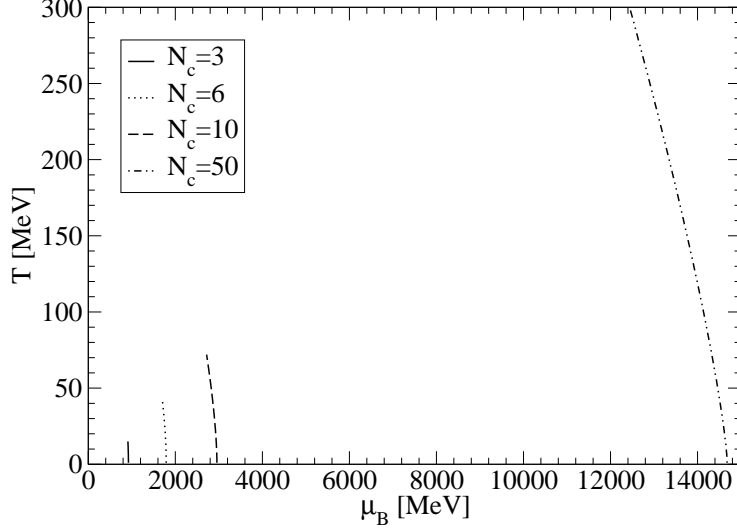


Figure 4: Liquid-gas transition critical lines for different values of the number of colors ( $N_c = 3, 6, 10, 50$ ).

It is interesting to observe that  $\Gamma_{\chi \rightarrow \bar{Q}Q}$  first increases as function of  $N_c$  in virtue of the increasing phase space, then it starts to decrease exponentially because of the suppressed decay amplitude. The interaction of this generalized tetraquark with quark-antiquark mesons assures that this object is well defined in the large  $N_c$  limit, but we will not need the scaling of Eq. (11).

(ii) The decay of the state  $\chi$  into two baryons takes place upon creation of a single quark-antiquark pair from the vacuum. Thus, the amplitude of this process is proportional to  $p$ , i.e. to  $N_c^0$ . The coupling constant  $g_\chi$  for the  $\chi$ -nucleons interaction scales as  $N_c^0$ .

We now repeat the study of the Walecka model by replacing the scalar state  $\sigma$  with the tetraquark state  $\chi$ . To this end we consider the Walecka Lagrangian

$$\begin{aligned} \mathcal{L} = & \bar{\psi}[\gamma^\mu(i\partial_\mu - g_\omega\omega_\mu) - (m_N - g_\chi\chi)]\psi + \frac{1}{2}\partial^\mu\chi\partial_\mu\chi - \frac{1}{2}m_\chi^2\chi^2 \\ & - \frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \frac{1}{2}m_\omega^2\omega^\mu\omega_\mu, \end{aligned} \quad (12)$$

with the following modified scaling properties:

$$m_\chi \rightarrow m_\chi \frac{2N_c - 2}{4}, \quad g_\chi \rightarrow g_\chi. \quad (13)$$

The numerical results are shown in Fig. (5), where we plot the binding energy per nucleon as a function of baryon density for different values of  $N_c$ . As it is clear from the picture, the tetraquark scenario leads to completely different results, compared with the previous scenario. In particular, the nuclear matter binding energy decreases so fast with  $N_c$  that already for  $N_c = 4$  no nuclear



matter exists. This is a consequence of the fact that, in this case, the medium-range attraction between nucleons is mediated by the exchange of a tetraquark meson. When the number of colors is increased, the tetraquark disappears, leading to a strong weakening of the attraction between the nucleons.

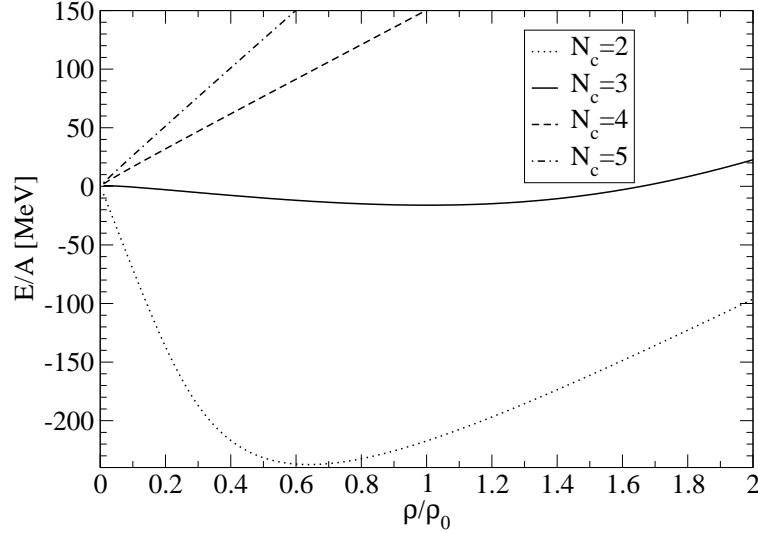


Figure 5: Binding energy per nucleon as a function of baryon density for the case in which the light quark-antiquark  $\sigma$  meson is replaced by a tetraquark state. In this case it is clear that nuclear matter is unbound already for  $N_c = 4$ .

### 2.5. The scenario with two scalar fields

Even if the lightest scalar state  $f_0(600)$  is mainly a tetraquark, one still expects a chiral partner of the pion above 1 GeV which can be interpreted as the resonance  $f_0(1370)$  [14]. In the literature scenarios with two scalar nonets have been investigated [18, 19]. Interestingly, in the detailed study of the nucleon-nucleon scattering performed in Ref. [20] two-scalar isoscalar states with masses  $m_{\sigma_1} \sim 400\text{-}600$  MeV and  $m_{\sigma_2} \sim 1200$  MeV have been considered. In view of the previous discussion we interpret the light scalar as predominantly tetraquark and the heavier one as predominantly quarkonium, see also Ref. [30] in which the potentially important role of a light tetraquark field at nonzero temperature has been outlined.

The natural question is if and how the present study is modified by the presence of two scalar-isoscalar states. We thus consider an extended version of the Walecka model with two scalar fields:

$$\begin{aligned} \mathcal{L} = & \bar{\psi}[\gamma^\mu(i\partial_\mu - g_\omega\omega_\mu) - (m_N - g_\sigma\sigma - g_\chi\chi)]\psi + \frac{1}{2}\partial^\mu\sigma\partial_\mu\sigma - \frac{1}{2}m_\sigma^2\sigma^2 \\ & + \frac{1}{2}\partial^\mu\chi\partial_\mu\chi - \frac{1}{2}m_\chi^2\chi^2 - \frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \frac{1}{2}m_\omega^2\omega^\mu\omega_\mu. \end{aligned} \quad (14)$$

It is interesting to notice that the general chiral model in Ref. [31] reduces to the Walecka form with two scalar fields in Eq. (14) when nuclear matter properties are investigated and when the glueball is neglected.

We use the following numerical values of the parameters:  $g_\chi^2/4\pi = 4.25$  with  $m_\chi = 470$  MeV,  $g_\sigma^2/4\pi = 17.61$  with  $m_\sigma = 1225$  MeV and  $g_\omega^2/4\pi = 14.28$  with  $m_\omega = 781$  MeV. The quantities  $g_\chi$ ,  $g_\sigma$  and  $m_\sigma$  are directly taken from Ref. [20]. The parameters  $m_\chi$  and  $g_\omega$  have been fixed to obtain saturation at  $\rho_0 = 0.16$  fm<sup>-3</sup> and an energy per nucleon  $E/A = -16$  MeV. Interestingly,  $m_\chi = 470$  MeV is only slightly changed with respect to the value of 452 MeV reported in Ref. [20], while  $g_\omega$  is 15% smaller than the value Ref. [20]. (In view of the large uncertainty on the coupling  $g_\omega$  this is not a large deviation, see also the discussion in Ref. [32]).

We then perform the large  $N_c$  study in accordance with Eqs. (3), (4), (5) and (13). The numerical results, depicted in Fig. 5, are quite similar to those obtained in the previous scenario. Although the heavy quark-antiquark state does not disappear with increasing  $N_c$ , its role alone is not sufficient to bind nucleons together: once again, the increase of  $N_c$  destroys nuclear matter. It should be stressed that our results do not depend on fine tuning: it is much more the nuclear matter for  $N_c = 3$  which follows from the detailed balance.

## 2.6. The scalar field as an effective treatment of TPE processes

The one-pion-exchange (OPE) has not been considered up to now because it does not contribute to the mean field approximation. Moreover, while the OPE is surely important for the long-range attraction between two nucleons, it is not enough to bind nuclei, see the review in Ref. [33] and the explicit study on the deuterium in Ref. [34] and refs. therein. A middle-range attraction mediated by a scalar particle is a necessary ingredient to describe nuclear matter.

However, one can go a step further and consider processes involving two-pion-exchange [21, 23], which generate the middle-range attraction in the scalar-isoscalar channel. The description through a scalar particle is then only an effective way to describe a process in which two pions are exchanged between two nucleons.

In order to study how these contributions behave in the large  $N_c$  limit it is first necessary to review the OPE and its properties for  $N_c \gg 3$ . The simplest OPE interaction term is given by:

$$\mathcal{L}_{1,\pi} = -g_\pi^{(1)} \bar{\pi} \psi \gamma^5 \vec{\tau} \psi, \quad (15)$$

Although  $g_\pi^{(1)} \propto \sqrt{N_c}$  in the large  $N_c$  limit, the OPE-potential does not scale as  $\left(g_\pi^{(1)}\right)^2 \propto N_c$ , but is proportional to  $\left(g_\pi^{(1)}\right)^2 / M_N^2 \propto N_c^{-1}$  and then it is suppressed when  $N_c \gg 3$ . This is due to the fact that, in virtue of the matrix  $\gamma^5$ , an additional factor  $1/M_N$  is associated to the emission of one pion. Notice that such a suppression is not present in the vector and scalar channels: the corresponding potentials scale as  $g_\omega^2 \propto N_c$  and  $g_{scalar}^2$ , whose scaling behavior depend on the assignment for the scalar field, see the discussion above.

Another OPE interaction is however possible and involves the derivative of the pion field:

$$\mathcal{L}_{2,\pi} = -g_\pi^{(2)} (\partial_\mu \vec{\pi}) \bar{\psi} \gamma^\mu \gamma^5 \vec{\tau} \psi . \quad (16)$$

Unlike the previous case, the corresponding OPE-potential is simply proportional to  $\left(g_\pi^{(2)}\right)^2$ . It is then crucial to understand how  $g_\pi^{(2)}$  scales in the large  $N_c$  limit. If  $g_\pi^{(2)}$  scales, as naively expected, as  $\sqrt{N_c}$ , then such a contribution to the OPE-potential survives for  $N_c \gg 3$ .

From the perspective of soft-pion emission, the Lagrangian  $\mathcal{L}_{1,\pi}$  in Eq. (15) can be equivalently replaced by a Lagrangian of the form of Eq. (16) in the following way:

$$\mathcal{L}_{1,\pi} \rightarrow -\frac{g_\pi^{(1)}}{2M_N} (\partial_\mu \vec{\pi}) \bar{\psi} \gamma^\mu \gamma^5 \vec{\tau} \psi . \quad (17)$$

Clearly, the large  $N_c$  behavior is not changed, being still  $\left(g_\pi^{(1)}/M_N\right)^2 \propto N_c^{-1}$ .

In linear sigma models without (axial-)vector mesons only the pion-nucleon interaction of the form given in Eq. (15) is present. Moreover, neglecting the small contribution from the nonzero current quark masses, the nucleon mass takes the form  $M_N = g_\pi^{(1)} f_\pi$ , thus one obtains:

$$\mathcal{L}_{1,\pi} \rightarrow -\frac{1}{2f_\pi} (\partial_\mu \vec{\pi}) \bar{\psi} \gamma^\mu \gamma^5 \vec{\tau} \psi , \quad (18)$$

where the scaling is not changed since  $f_\pi \propto \sqrt{N_c}$ .

In general, however, the full pion-nucleon interaction in the soft-pion limit takes the form:

$$\mathcal{L}_{\pi,full} = \mathcal{L}_{1,\pi} + \mathcal{L}_{2,\pi} \rightarrow -\left(\frac{g_\pi^{(1)}}{2M_N} + g_\pi^{(2)}\right) (\partial_\mu \vec{\pi}) \bar{\psi} \gamma^\mu \gamma^5 \vec{\tau} \psi . \quad (19)$$

In chiral perturbation theory [21], which is also defined in the soft-pion limit, the following pion-nucleon interaction term is present:

$$\mathcal{L}_\pi^{chPT} = -\frac{g_A}{2f_\pi} (\partial_\mu \vec{\pi}) \bar{\psi} \gamma^\mu \gamma^5 \vec{\tau} \psi \quad (20)$$

where  $g_A$  is the axial-coupling of the nucleon. By comparison we find

$$g_A = 2f_\pi \left(\frac{g_\pi^{(1)}}{2M_N} + g_\pi^{(2)}\right) = 1 + 2f_\pi g_\pi^{(2)}. \quad (21)$$

In the already mentioned sigma models without vector mesons the constant  $g_\pi^{(2)} = 0$ , and one has therefore  $g_A = 1 \propto N_c^0$ . The situation changes, however, when vector mesons are included: in those models  $g_\pi^{(2)}$  does not vanish and scales as  $\sqrt{N_c}$ . This, in turn, implies that  $g_A \propto N_c$ . The reason why a term of the kind  $g_\pi^{(2)} (\partial_\mu \vec{\pi}) \bar{\psi} \gamma^\mu \gamma^5 \vec{\tau} \psi$  with  $g_\pi^{(2)} \propto \sqrt{N_c}$  emerges is rather subtle: it

follows from the so-called  $a_1$ - $\pi$  mixing, see Ref. [31] for details. On a numerical level, in these generalized sigma models the following large  $N_c$  scaling for the axial-coupling constant  $g_A$  is obtained:

$$g_A(N_c) = 1 + 0.267 \frac{N_c}{3} = 1 + (g_A^{\text{exp}} - 1) \frac{N_c}{3} \quad (22)$$

where  $g_A^{\text{exp}} = 1.267 \pm 0.004$ . There is therefore a factor 1 which is large  $N_c$  independent and a factor  $(g_A^{\text{exp}} - 1) N_c/3 = 0.267 N_c/3$ , which is subdominant for  $N_c = 3$  but which dominates for  $N_c$  large enough<sup>1</sup>. Thus, the deviation of the axial coupling constant from unity becomes dominant in the large  $N_c$  limit. This discussion shows that a OPE-term does not disappear in the large  $N_c$  limit, although its strength with respect to the  $\omega$  meson repulsion is smaller than in the vacuum.

We now turn to the TPE case, which can mimic the middle-range scalar attraction. When considering two pions as intermediate state, the naive scaling of the corresponding TPE potential is proportional to  $(g_A/2f_\pi)^2 \propto N_c^2$ . This result is however not correct: the  $N_c^2$  contribution from the box diagrams cancel with the  $N_c^2$  contribution from the crossed-box diagram, see Ref.[36] for details. The resulting TPE potential is then proportional to  $(g_A/2f_\pi)^2/N_c \propto N_c$ .

Now, when substituting the TPE interactions with an effective scalar field  $\sigma_{TPE}$  (where the effective nature of this field, in comparison to the quarkonium and tetraquark cases, should be clear) we have the following Walecka-type Lagrangian

$$\begin{aligned} \mathcal{L} = & \bar{\psi}[\gamma^\mu(i\partial_\mu - g_\omega\omega_\mu) - (m_N - g_{\sigma_{TPE}}\sigma_{TPE})]\psi + \frac{1}{2}\partial^\mu\sigma_{TPE}\partial_\mu\sigma_{TPE} \\ & - \frac{1}{2}m_{\sigma_{TPE}}^2\sigma_{TPE}^2, \end{aligned} \quad (23)$$

with

$$g_{\sigma_{TPE}}^2(N_c) = \left[ \frac{g_A(N_c)}{g_A(N_c=3)} \right]^2 \frac{3g_{\sigma_{TPE}}^2(N_c=3)}{N_c} \quad (24)$$

and

$$m_{\sigma_{TPE}}^2 \sim 4m_\pi^2 \propto N_c^0. \quad (25)$$

Notice that  $g_{\sigma_{TPE}}^2(N_c)$  is chosen in such a way that, as usual, for  $N_c = 3$  the value  $g_{\sigma_{TPE}}^2(N_c=3)/4\pi = 2.418$  necessary for the experimentally observed saturation is realized.

In order to test if nuclear matter exist for large  $N_c$  in this scenario we repeat our study by using the new scaling in Eqs. (24) and (25). Although  $g_{\sigma_{TPE}}$  scales

---

<sup>1</sup>When the so called mirror assignment for the nucleon and its chiral partner is considered in the framework of the linear sigma models [31, 35], Eq. (22) changes as  $g_A(N_c) = a + bN_c/3$ , where  $a$  is not necessarily unity. However, the large  $N_c$  behavior is unaffected and, using the numerical results of Ref. [31], the result  $g_A(N_c) = 0.93 + 0.33N_c/3$  is obtained, which is only slightly changed w.r.t. Eq. (22). All the conclusions presented in this section hold therefore also in the mirror assignment.

with  $N_c$  (as in the quarkonium assignment), no saturation is obtained in the large  $N_c$  limit. The reason for this result is that the ratio  $g_{\sigma_{TPE}}/g_\omega$  decreases as soon as  $N_c = 3$  is left and then approaches a constant for  $N_c \rightarrow \infty$ , which is however smaller than the value  $(g_{\sigma_{TPE}}/g_\omega)_{N_c=3}$ . For instance, we can obtain a stable nuclear matter for  $N_c = 4$  (but not for larger values of  $N_c$ ) only if we would –artificially– use a large value  $g_A^{\text{exp}} \gtrsim 5$ . This is, however, not the case in our world, where  $g_A^{\text{exp}} = 1.267$ .

We thus conclude that, also when the scalar attraction is mediated by TPE processes in the scalar channel, nuclear matter does not bind for large  $N_c$ . This result holds true even when  $g_A$  scales as  $N_c$ , provided that the axial coupling constant measured in the real world for  $N_c = 3$  is reproduced.

### 2.7. Further scenarios

- Dilaton/Glueball: In this work we did not consider the glueball field as a possible intermediate boson for the nucleon-nucleon interaction. Although potentially important in dilatation invariant models and for the scalar phenomenology [37], the mass of the glueball is about 1.5 GeV [38] and is too high to affect nuclear matter binding. There are however models in which the lightest scalar resonance  $f_0(600)$  is interpreted as a glueball state, e.g. [39] and refs. therein. Although we consider this assignment unfavored due to the too low glueball mass in comparison with the lattice value, for completeness we perform a study of this scenario. The corresponding Lagrangian reads

$$\begin{aligned} \mathcal{L} = & \bar{\psi}[\gamma^\mu(i\partial_\mu - g_\omega\omega_\mu) - (m_N - g_G G)]\psi + \frac{1}{2}\partial^\mu G\partial_\mu G - \frac{1}{2}m_G^2 G^2 \\ & - \frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \frac{1}{2}m_\omega^2\omega^\mu\omega_\mu . \end{aligned} \quad (26)$$

The leading order large  $N_c$  scaling relations are given by

$$m_G \longrightarrow m_G , \quad g_G \rightarrow g_G , \quad (27)$$

i.e. they are both large  $N_c$  invariant. When repeating the study in this case no bound state in the large  $N_c$  scenario exists.

- In many works on light hadron states it is found that light scalars are ‘dynamically generated’ [12], see also the discussion about dynamically generated and reconstructed states in Ref. [11]. In particular, the light resonance  $f_0(600)$  does not correspond to any of the previously analyzed cases (quark-antiquark, tetraquark or glueball) but is a pion-pion ‘molecular’ bound state. Notice that the here analyzed scenario is also different from the TPE case studied in Sec. 2.7, where two pions were simultaneously exchanged between two nucleons, without interacting with each other. Although the emission of the two pions is necessary in the present case, the crucial point here is their further interaction to generate a new resonance. This new resonance, contrary to the simple TPE processes, definitely disappears in the large  $N_c$  limit. The reason is that the attraction

in the  $\pi\pi$  channel is mediated by meson exchange (such as the exchange of a  $\rho$  meson), whose corresponding scattering amplitudes scale as  $1/N_c$ . For  $N_c$  large enough the interaction strength fades out and the light scalar resonance ceases to exist. Thus, even its effect for nuclear matter does not take place: no binding at large  $N_c$  takes place in scenarios in which  $f_0(600)$  is dynamically generated.

- The previous conclusion holds also when the light resonance  $f_0(600)$  emerges as a low-energy companion pole [13]. In fact, these new poles disappear for  $N_c \gg 3$ .

### 3. Conclusions

In this work we studied the formation of nuclear matter for  $N_c \gg 3$ . We conclude that the present phenomenological information about scalar mesons implies that no nuclear matter exists for large  $N_c$ . In fact, the only case in which nuclear matter does not disappear by increasing  $N_c$  is the naive quarkonium assignment for the lightest scalar resonance. This scenario is criticized by many recent and less recent studies of low-energy hadron phenomenology, which agree that the light scalar states below 1 GeV are not predominantly quarkonium states. Moreover, even in the quarkonium picture one should at least include the effects of the pion-pion dressing, which is expected to be large in view of the broad nature of the resonance. This property is enough to ‘unbind’ nuclear matter for large  $N_c$ .

The non existence of nuclear matter for large  $N_c$  has been explicitly shown in alternative scenarios for the light scalar states. We first concentrated on the tetraquark interpretation, in which a peculiar large  $N_c$  limit has been discussed. We then studied the cases in which the nucleon-nucleon interaction in the scalar channel is dominated by: (i) two scalar fields, (ii) TPE processes, (iii) a glueball state, and (iv) a pion-pion molecular state. The common feature of all these assignments is that nuclear matter does not bind for large  $N_c$ . Numerically, the value  $N_c = 4$  is already enough to render nuclear matter unstable.

The results of this work have been derived by using Walecka-type models. We have limited the study to nuclear matter density and small temperatures, where the Walecka model represents a well-defined and useful theoretical tool. Moreover, the main goal of the present work is not a precise numerical study of nuclear matter properties, but simply to assess its existence for  $N_c \gg 3$ : for this reason a simple and schematic model as the Walecka one fulfills the desired requirements. However, it is surely an interesting task for the future to repeat the present study going beyond the mean field approximations used here.

Obviously we do live in a world in which nuclear matter exists. The laws of Nature must allow for nuclear matter, so that life as we know it might evolve (anthropic principle). The subtle point is not (only) the existence of nuclear matter, but the fact that the binding energy per nucleon  $E_B \simeq 16$  MeV is much smaller than the natural scale of the system,  $\Lambda_{QCD} \simeq 200$  MeV. Our outcome that nuclear matter exists only for  $N_c \lesssim 3$ , and is thus a peculiar property

of our world, is in agreement with the phenomenological realized smallness of the ratio  $E_B/\Lambda_{QCD} \simeq 0.1$ . In fact, in the tetraquark (or molecular) scenario, the decreasing of  $N_c$  favors the formation of nuclear matter. By decreasing  $N_c$  from 3 to 2 an increase of the binding energy is obtained. In the framework of nuclear matter,  $N_c = 3$  is not large at all. It should be stressed that all this is not true when the lightest scalar state is a quarkonium state, for which the relation  $E_B/\Lambda_{QCD} \sim N_c$  holds. In this (unfavored) scenario the smallness of the binding energy could not be understood. Note that, while it is clearly not possible to investigate experimentally the (non)binding of nuclear matter for  $N_c > 3$ , this can be the subject of computer simulations of QCD, in which the number of colors is a parameter which can be easily changed.

Many studies have already shown that the conditions for the existence of complex life represent a small volume in the space of the free parameters (coupling constants and masses) of the Standard Model (e.g. [40] and refs. therein). The present study shows that these conditions are restricted also in the direction of  $N_c$ . The change of  $N_c$  can be regarded as a change of the group structure of the standard model. The fact that the group of the strong interaction in our Universe is  $SU(N_c)$  with  $N_c \lesssim 3$  should not be a surprise.

**Acknowledgement:** The authors thank Daniel Fernandez-Freile, G. Pagliara, A. Heinz, G. Torrieri and S. Lottini for useful discussions. L. B. is supported by the Hessen Initiative for Excellence (LOEWE) through the Helmholtz International Center for FAIR (HIC for FAIR).

## References

- [1] G. 't Hooft, Nucl. Phys. B **72** (1974) 461.
- [2] E. Witten, Nucl. Phys. B **160** (1979) 57.
- [3] S. R. Coleman, “1/N,” Published in Erice Subnuclear 1979:0011. R. F. Lebed, Czech. J. Phys. **49** (1999) 1273 [arXiv:nucl-th/9810080]. E. E. Jenkins, Ann. Rev. Nucl. Part. Sci. **48** (1998) 81 [arXiv:hep-ph/9803349].
- [4] L. McLerran and R. D. Pisarski, Nucl. Phys. A **796** (2007) 83 [arXiv:0706.2191 [hep-ph]].
- [5] L. McLerran, K. Redlich and C. Sasaki, arXiv:0812.3585 [hep-ph]. C. Sasaki and I. Mishustin, Phys. Rev. C **82** (2010) 035204 [arXiv:1005.4811 [hep-ph]].
- [6] G. Torrieri and I. Mishustin, Phys. Rev. C **82** (2010) 055202 [arXiv:1006.2471 [nucl-th]]. G. Torrieri and I. Mishustin, arXiv:1101.0149 [nucl-th].

- [7] Y. Hidaka, T. Kojo, L. McLerran and R. D. Pisarski, arXiv:1004.2261 [hep-ph].
- [8] T. Kojo, Y. Hidaka, L. McLerran and R. D. Pisarski, Nucl. Phys. A **843** (2010) 37 [arXiv:0912.3800 [hep-ph]].
- [9] J. D. Walecka, Annals Phys. **83**, 491 (1974). B. D. Serot and J. D. Walecka, Adv. Nucl. Phys. **16**, 1 (1986). B. D. Serot and J. D. Walecka, Int. J. Mod. Phys. E **6**, 515 (1997) [arXiv:nucl-th/9701058].
- [10] C. Amsler and N. A. Tornqvist, Phys. Rept. **389**, 61 (2004). E. Klempt and A. Zaitsev, Phys. Rept. **454** (2007) 1 [arXiv:0708.4016 [hep-ph]].
- [11] F. Giacosa, Phys. Rev. D **80** (2009) 074028 [arXiv:0903.4481 [hep-ph]].
- [12] J. R. Pelaez, Phys. Rev. Lett. **92** (2004) 102001. Mod. Phys. Lett. A **19** (2004) 2879. M. Uehara, arXiv:hep-ph/0401037. J. A. Oller and E. Oset, Nucl. Phys. A **620** (1997) 438 [Erratum-ibid. A **652** (1999) 407] [arXiv:hep-ph/9702314]. F. Giacosa and G. Pagliara, Nucl. Phys. A **833** (2010) 138 [arXiv:0905.3706 [hep-ph]].
- [13] M. Boglione and M. R. Pennington, Phys. Rev. D **65** (2002) 114010 [arXiv:hep-ph/0203149]. E. van Beveren, T. A. Rijken, K. Metzger, C. Dullemond, G. Rupp and J. E. Ribeiro, Z. Phys. C **30** (1986) 615 [arXiv:0710.4067 [hep-ph]].
- [14] D. Parganlija, F. Giacosa and D. H. Rischke, Phys. Rev. D **82** (2010) 054024 [arXiv:1003.4934 [hep-ph]].
- [15] R. L. Jaffe, Phys. Rev. D **15** (1977) 267. R. L. Jaffe, Phys. Rev. D **15** (1977) 281.
- [16] R. L. Jaffe, Phys. Rept. **409** (2005) 1 [Nucl. Phys. Proc. Suppl. **142** (2005) 343] [arXiv:hep-ph/0409065].
- [17] L. Maiani, F. Piccinini, A. D. Polosa and V. Riquer, Phys. Rev. Lett. **93** (2004) 212002 [arXiv:hep-ph/0407017]. F. Giacosa, Phys. Rev. D **74** (2006) 014028 [arXiv:hep-ph/0605191].
- [18] A. H. Fariborz, R. Jora and J. Schechter, Phys. Rev. D **72** (2005) 034001 [arXiv:hep-ph/0506170]. A. H. Fariborz, Int. J. Mod. Phys. A **19** (2004) 2095. [arXiv:hep-ph/0302133]. M. Napsuciale and S. Rodriguez, Phys. Rev. D **70** (2004) 094043.
- [19] F. Giacosa, Phys. Rev. D **75** (2007) 054007 [arXiv:hep-ph/0611388].
- [20] R. Machleidt, Phys. Rev. C **63** (2001) 024001 [arXiv:nucl-th/0006014].
- [21] N. Kaiser, R. Brockmann and W. Weise, Nucl. Phys. A **625** (1997) 758 [arXiv:nucl-th/9706045]. N. Kaiser, S. Gerstendorfer and W. Weise, Nucl. Phys. A **637** (1998) 395 [arXiv:nucl-th/9802071]. L. Girlanda, A. Rusetsky and W. Weise, Annals Phys. **312** (2004) 92 [arXiv:hep-ph/0311128].



- [22] D. B. Kaplan and A. V. Manohar, Phys. Rev. C **56** (1997) 76 [arXiv:nucl-th/9612021]. M. M. Kaskulov and H. Clement, Phys. Rev. C **70** (2004) 014002 [arXiv:nucl-th/0401061]. S. R. Beane, arXiv:hep-ph/0204107.
- [23] A. Calle Cordon and E. Ruiz Arriola, Phys. Rev. C **80** (2009) 014002 [arXiv:0904.0421 [nucl-th]]. A. Calle Cordon and E. Ruiz Arriola, Phys. Rev. C **81** (2010) 044002 [arXiv:0905.4933 [nucl-th]].
- [24] B.W. Lee, “Chiral Dynamics”, Gordon and Breach, New York, 1972.
- [25] S. Gasiorowicz and D. A. Geffen, Rev. Mod. Phys. **41** (1969) 531.
- [26] Y. Nambu and G. Jona-Lasinio, Phys. Rev. **122** (1961) 345. For reviews: T. Hatsuda and T. Kunihiro, Phys. Rept. **247**, 221 (1994) [arXiv:hep-ph/9401310]. S. P. Klevansky, Rev. Mod. Phys. **64** (1992) 649.
- [27] F. Giacosa and G. Pagliara, Phys. Rev. C **76** (2007) 065204 [arXiv:0707.3594 [hep-ph]].
- [28] A. De Rujula, H. Georgi and S. L. Glashow, Phys. Rev. D **12** (1975) 147. T. A. DeGrand, R. L. Jaffe, K. Johnson and J. E. Kiskis, Phys. Rev. D **12** (1975) 2060. G. 't Hooft, Phys. Rev. D **14**, 3432 (1976) [Erratum-ibid. D **18**, 2199 (1978)]. E. V. Shuryak, Nucl. Phys. B **203** (1982) 93. T. Schafer and E. V. Shuryak, Rev. Mod. Phys. **70** (1998) 323 [arXiv:hep-ph/9610451]. E. Shuryak and I. Zahed, Phys. Lett. B **589** (2004) 21 [arXiv:hep-ph/0310270]. U. Vogl and W. Weise, Prog. Part. Nucl. Phys. **27** (1991) 195. P. Maris and C. D. Roberts, Int. J. Mod. Phys. E **12** (2003) 297 [arXiv:nucl-th/0301049].
- [29] C. Liu, Eur. Phys. J. C **53** (2008) 413 [arXiv:0710.4185 [hep-ph]].
- [30] A. Heinz, S. Struber, F. Giacosa and D. H. Rischke, Phys. Rev. D **79** (2009) 037502 [arXiv:0805.1134 [hep-ph]].
- [31] S. Gallas, F. Giacosa and D. H. Rischke, Phys. Rev. D **82** (2010) 014004 [arXiv:0907.5084 [hep-ph]].
- [32] R. Machleidt, K. Holinde and C. Elster, Phys. Rept. **149** (1987) 1.
- [33] M. Baldo and G. F. Burgio, arXiv:1102.1364 [nucl-th].
- [34] Y. B. Ding *et al.*, J. Phys. G **30** (2004) 841 [arXiv:hep-ph/0402109].
- [35] C. DeTar and T. Kunihiro, Phys. Rev. D **39** (1989) 2805.
- [36] M. K. Banerjee, T. D. Cohen and B. A. Gelman, Phys. Rev. C **65** (2002) 034011 [arXiv:hep-ph/0109274].

- [37] C. Amsler and F. E. Close, Phys. Rev. D **53** (1996) 295 [arXiv:hep-ph/9507326]. W. J. Lee and D. Weingarten, Phys. Rev. D **61**, 014015 (2000). [arXiv:hep-lat/9910008]; F. E. Close and A. Kirk, Eur. Phys. J. C **21**, 531 (2001). [arXiv:hep-ph/0103173]. F. Giacosa, T. Gutsche, V. E. Lyubovitskij and A. Faessler, Phys. Rev. D **72**, 094006 (2005). [arXiv:hep-ph/0509247]. F. Giacosa, T. Gutsche, V. E. Lyubovitskij and A. Faessler, Phys. Lett. B **622**, 277 (2005) [arXiv:hep-ph/0504033]. F. Giacosa, T. Gutsche and A. Faessler, Phys. Rev. C **71**, 025202 (2005) [arXiv:hep-ph/0408085]. H. Y. Cheng, C. K. Chua and K. F. Liu, Phys. Rev. D **74** (2006) 094005 [arXiv:hep-ph/0607206]. L. Bonanno and A. Drago, Phys. Rev. C **79**, 045801 (2009) [arXiv:0805.4188 [nucl-th]].
- [38] Y. Chen *et al.*, Phys. Rev. D **73** (2006) 014516. [arXiv:hep-lat/0510074].
- [39] V. Mathieu, N. Kochelev and V. Vento, Int. J. Mod. Phys. E **18** (2009) 1 [arXiv:0810.4453 [hep-ph]].
- [40] R. L. Jaffe, A. Jenkins and I. Kimchi, Phys. Rev. D **79** (2009) 065014 [arXiv:0809.1647 [hep-ph]]. R. Kallosh and A. D. Linde, Phys. Rev. D **67** (2003) 023510 [arXiv:hep-th/0208157]. V. Agrawal, S. M. Barr, J. F. Donoghue and D. Seckel, Phys. Rev. D **57** (1998) 5480 [arXiv:hep-ph/9707380].